

The Distance to the Virgo Cluster

The aims of this experiment are to carry out aperture photometry of Cepheid variable stars observed in the Virgo-cluster galaxy M100, to construct light curves for the variables and use the period-luminosity relationship for Cepheids to determine the distance to M100. Given the recession velocity of the Virgo cluster, a value for the Hubble constant is derived.

1 Introduction

The precise determination of the distances to galaxies is not easy. The inter-galactic distances are so vast that the method of trigonometric parallax used for determining fundamental stellar distances is totally inadequate. The methods of galactic distance determination are indirect, relying largely on the properties of the brighter stars contained in galaxies. By comparing the known absolute magnitude of a distance indicator with the measured apparent magnitude of the same type of object in another galaxy, the distance to the galaxy can be determined using the distance modulus relation.

A powerful tool for distance determination is the period-luminosity relationship of Cepheid variables: the relationship between a Cepheid's period of variation and its intrinsic luminosity has been calibrated by observing Cepheids in the Large Magellanic Cloud (LMC, a companion galaxy to the Milky Way), whose distance has been measured by other means. In principle, by measuring the apparent magnitude and period of variation of any Cepheid, its intrinsic luminosity can be obtained and hence its distance. It was by recognising the tell-tale variation of Cepheids in the Andromeda nebula that Hubble was first able to establish beyond all doubt that it was an independent spiral galaxy outside of the Milky Way.

The Cepheid period-luminosity relationship is one of the most important fundamental methods for determining the distances to extragalactic objects. However, although some Cepheids are very bright, their variation can only reliably be detected up to a few megaparsecs (Mpc) away from ground-based observatories. The detection limit of the Hubble Space Telescope (HST) is much fainter, and one of the key projects for the HST was to obtain "Cepheid distances" to galaxies within some of the nearby clusters of galaxies. The distances obtained provide not only a local estimate of the Hubble constant, but also permit calibrations of brighter, so-called secondary distance indicators (such as supernovae), which can then be used to determine the distances of more remote clusters of galaxies in the Universe; and so we climb further up the distance-scale ladder. For background reading, consult reference 1.

1.1 The experiment

In this exercise, you will examine the actual HST data which was used to measure the distance to the galaxy M100, a member of the Virgo cluster. You will measure the apparent magnitudes of several Cepheids within M100 at various dates over a two-month period. You will then plot light-curves for the Cepheids and attempt to obtain an

estimate of their periods of variation and mean apparent magnitudes. From the period-luminosity relation, you will then derive the distance modulus, and hence the distance, to M100, and determine a value for the Hubble constant from the Virgo-cluster recession velocity.

2 Measuring the Cepheid magnitudes

The refurbished HST observed the galaxy M100 on 12 occasions over a two-month period between 1994 April and June—the calendar and Julian dates are given in Table 1. Observations were made with the second-generation Wide-Field and Planetary Camera (WFPC2) through a filter which approximates to the *V*-band of the *UBV* system.

Table 1: Dates of HST observations of M100

Calendar Date 1994	Julian Date (−2449000)
Apr 23	465.781
May 04	476.707
May 06	478.986
May 09	482.401
May 12	485.216
May 16	489.037
May 20	493.528
May 26	498.824
May 31	503.852
Jun 07	510.819
Jun 17	520.949
Jun 19	522.959

Cepheids were identified by obtaining (with a fair degree of automation!) photometric measurements of all of the stellar objects in the field for each date and searching the sample of data for the signature of Cepheid variability (Ferrarese *et al.*, 1996 — ref. 3).

2.1 Measuring procedure

The measurements are made using a program running on the Unix system at ULO. A demonstrator will provide you with access to an account you can use for this exercise. Once logged in, the program is started by typing `cepheids`. A window should appear for selecting, displaying, and measuring the data.

For orientation, the initial image displayed is the full WFPC2 field of view for one particular date. The cepheids selected for this exercise are located in the upper right quadrant of this field; again, for orientation, you can see the location of all the identified cepheids in the field from the hard-copy image of this quadrant. Small sections of images, containing some of the Cepheids identified in this quadrant, have been prepared for you to make measurements on.

The stars will be measured using a technique known as *aperture photometry*; a small aperture is located on a star in the digital image and the amount of light within that aperture is measured, by adding up the CCD counts enclosed. This simulates the technique which has been carried out on ground-based telescopes for many years, using

physical apertures to isolate stars for measurement with a photomultiplier. Of course, the aperture may enclose background light too, so some means of estimating and subtracting its contribution must be found. In the method described below, this will usually be done by measuring the sky background light in a small concentric annulus around the stellar aperture.

To make a full set of measurements for one Cepheid, follow through these steps:

1. Select a Cepheid number (start with no. 4) and the date (start with Apr 23); the cepheid selection determines which section of the full image obtained on that date will be displayed.
2. Set some parameters to control the image display: you should set the percentiles for display to be 1 and 99 to begin with. You can adjust these later to suit: raising the lower percentile will make the background darker; lowering the higher percentile will make the highlights brighter. These values only affect the display, not the measurement.
3. Check the parameters for the photometry—the default values should be adequate, but you can change some if necessary:
 - (a) The default radius of the star aperture is 2 pixels. This can not be changed; it must be kept the same for *all* the measurements.
 - (b) For sky background measurement, the default method is to use a concentric annulus around the star aperture. The default size of the annulus is set to be from $2\times$ to $3\times$ the star-aperture radius (*i.e.*, a ring with inner radius 4 pixels and outer radius 6 pixels). This should be adequate for most measurements.
4. Display the image by clicking the button `Display image . . .`. Identify the star to be measured from the finding charts provided (taken from ref. 3 — these charts are about 1/4 the size of the displayed image). In most (but not all) cases, the Cepheid will be found near the centre of the 100×100 -pixel image section.
 - Click the left-hand mouse button on the star to draw the aperture(s); the measured values will automatically be displayed. This can be repeated as often as you like until you have a measurement you are happy with.
 - Click the right-hand mouse button to finish measuring the frame. You can finish and redisplay the image at any time, to change the display percentiles, or to erase the apertures already drawn.
5. For each measurement, you should record the number of counts in the star and the number of sky background counts. The sky background given is the average count per pixel in the annulus; the star count given is the total count over the whole (small) aperture, *after subtraction of the mean sky background for each pixel within the aperture. Therefore, the star measurement has already been corrected for the sky background!*
6. Select the next date for the same Cepheid and follow steps 2–5 above.

You should make a series of measurements for the whole range of dates, for all six Cepheids. Tabulate your results neatly, recording the star signal and the mean sky background. Note that the exact value of a given measurement is sensitive to the exact location of the aperture; you probably do not have time to take an average of several

measurements, but you should at least try to find a way to estimate the likely uncertainty in the measurements (a few trial values will indicate how many significant figures to retain in the star and sky-average counts, for example).

Record any additional information which may help you later to decide whether a given measurement is reliable or not (*e.g.*, if neighbouring stars interfere badly with the measurement).

Note that the sky background measurements should at least be made consistently for the same star. If fainter background stars fall in the annulus, it should not matter too much as long as they do so consistently in each frame (but this may affect the calculation of the mean apparent magnitude later on, hence it should be noted). If nearby bright stars intrude into the background measurement, the annulus size can be changed to suit.

[Alternatively (but much more difficult to do repeatably), a separate estimate of the sky background can be made—to use this method, switch off the **concentric annulus** option before displaying the image. To measure a star, the sky background must first be sampled by clicking the *middle* mouse button (for two-button mice, this means press them both at the same time!) in a clear field next to the star; the star can then be measured as before with the left-hand mouse button. The right-hand button quits the measuring procedure for that frame.]

Q.1 You may notice bright pixels which appear randomly in some of the images; these are caused by cosmic rays which hit the detector during the exposure (most have already been “cleaned” off the images during processing, but some remain). How much do “cosmic-ray” pixels affect the sky value, when they occur in the annulus? Is this a significant problem for the purpose of the experiment?

2.2 Calculation of apparent magnitude

For these data, convert the star counts, N , to apparent magnitude, m , using the relation:

$$m = -2.5 \log_{10} N + 30.40, \quad (1)$$

where the addition of the constant reduces the data to standard magnitudes in the photometric V band. Reduce all your star data to obtain a table of dates and magnitude estimates for each star. Calculate also the mean counts for each star averaged over all dates, and convert these to a mean magnitude for each star, \bar{m} . (Note that you do not need to use the sky values in any calculations; but they may serve to help investigate the source of any discrepant data later on.)

3 Periods and distances

3.1 Light curve and period analysis

Use a spreadsheet package such as EXCEL to plot the light curves and to conduct the period analysis. (You may plot the curves by hand, and devise an alternative technique for estimating the period and error, if you wish, but using EXCEL will conveniently allow you to try out different test values for the period, as described below.) A reasonable estimate of the period and error can be achieved in this way; it is more rigorous to use analytical techniques, but this will not be attempted here.

1. Plot the light curve for each star separately as magnitude against Julian Date (see Table 1). (Note that the ordinate scale should be decreasing, since numerically smaller magnitudes mean brighter phases in the variation.)
2. A reasonable estimate of the period, P , for each star can be obtained by simple inspection of the light curve, *but note that not every star will show much more than one complete period of variation!* Estimate the period of variation by looking for repeated sections of the light curve, or repeated maxima/minima.
3. For each Cepheid, enter your period estimate into a spreadsheet cell (so that you can reference its value in the next step, and then adjust it as necessary later on).
4. For each star, *fold* the data on your estimated period by doing the following:
 - (a) Add your initial estimate of the period P to the dates of all the observations (*i.e.* with a formula which references the cell containing the period), to create a new series of data for that star. Then plot these new data on the light curve, using different symbols (EXCEL has a way of allowing you to add a second series of data to a plot — if you don't know how to do this, ask a demonstrator).
 - (b) Then adjust the period in the cell to try out different values, until the folded light curve shows a smooth cycle of variation (remember that the data points do have associated uncertainties, so there will always be some scatter).
 - (c) For Cepheids with shorter periods, you may find it useful to add a *third* series of data by adding $2P$ to the first series of data — in this way, at least part of the plot will show all the data obtained at the same *phase*, plotted in the same section of the light curve.
5. Hence, obtain refined estimates of the period for each Cepheid, and plot the six light curves showing the data folded on the final period. From the scatter of data in the folded light curve, and therefore the uncertainty in fixing the period precisely, obtain an estimate of the error on each period.

3.2 The period-luminosity relation

The absolute magnitude of a Cepheid is directly related to its period: brighter Cepheids have longer periods. This property, expressed in the period-luminosity relation, is the key to obtaining the absolute magnitudes of distant Cepheids, and hence their distances.

The period-luminosity relation used here is calibrated using observations of Cepheids in the Large Magellanic Cloud. The absolute magnitude, M_V , is given by:

$$M_V = -2.76 \log_{10} P - 1.40, \quad (2)$$

where P is the period of the Cepheid in days.

Calculate the absolute magnitudes of all six Cepheids, and hence obtain the apparent distance modulus for each star, $\bar{m} - M_V$, where \bar{m} is the mean apparent magnitude determined earlier. Obtain the mean apparent distance modulus of M100, and its error.

Assuming that the reddening of M100 is given by $E(B - V) = 0.10 \pm 0.02$, obtain the *true* distance modulus of M100. Hence derive the distance to M100, and its error.

(For the interested, you might wish to compare your results with the published values given in ref. 2; these were obtained using the very same data, but using more formal techniques of measurement and analysis.)

- Q.2** (a) What possible source of systematic error might arise from using equation (2)? [Think about how equation (2) is calibrated.]
(b) What other basic assumption is made by applying equation (2) to the Cepheids in M100?

3.3 The Hubble constant

A knowledge of the distance to M100 allows an estimate of the Hubble constant (H_0) to be made, if its recession velocity is known also. In fact, this is quite easy to measure from spectroscopic observations. However, some of the observed motion of M100 will be due not only to the Hubble flow, but also due to the galaxy's so-called peculiar motion within the Virgo cluster; it is therefore better to use the mean recession velocity of the whole Virgo cluster: $1396 \pm 96 \text{ km s}^{-1}$.

From your calculated distance to M100, obtain a value for the Hubble constant and its error (the typical units used are $\text{km s}^{-1} \text{ Mpc}^{-1}$, *i.e.*, the rate at which we see the Universe expanding, in units of km s^{-1} for each megaparsec of distance away from us.)

The *relative* (not absolute) distance between the Virgo cluster and the more-distant Coma cluster of galaxies is precisely known from studies of other distance indicators, and amounts to 3.71 ± 0.10 magnitudes in the (true) distance modulus. Since the Coma recession velocity of $7200 \pm 100 \text{ km s}^{-1}$ is more precisely known than for Virgo—the motion due to the Hubble flow dominates much more the galaxy peculiar motions in Coma—a more precise value of the Hubble constant can be derived. Calculate a value for H_0 and its error using the Coma cluster values.

- Q.3** What source of systematic error might arise from using the distance to M100 to derive these values of the Hubble constant? [Think about the physical size of the Virgo cluster itself (its angular size on the sky is several degrees).]

4 References

1. Zeilik & Gregory, Sections 22.3 and 18.2.
2. Ferrarese *et al.*, 1996. 'The discovery of Cepheids and a new distance to M100 using the Hubble Space Telescope', *Astrophys. J.*, vol. 464, 568.