

UNIVERSITY COLLEGE LONDON

University Of London Observatory

PHAS1510 – Certificate in Astronomy, 1213.01

PHAS1510-06: The Orbits of the Moons of Jupiter

Name: _____

An experienced student should aim to complete this practical in 2 (and not more than 3) sessions.

1 Objectives

The objectives of this experiment are to measure the orbital properties of Jupiter's moons, and thence to apply Kepler's Third Law of planetary motion to calculate the mass of Jupiter.

2 Items required

You will need:

- access to a PC running MS-Windows and the CLEA/ULO program 'The Revolution of the Moons of Jupiter', and a scientific pocket calculator. (Before running the CLEA/ULO program you should ensure that the MS-Windows default printer on the desktop of the ULO computer you are using is set to the 'ULO-NWcolour' printer. Ask a demonstrator to make the change for you if you are unsure how to do this.)

(This practical is also available on the Certificate's Web site.)

3 Introduction

*[You may skip this Introduction if you want to get on with the experiment immediately, but it is **essential** that you are familiar with, and able to use, equation 2.]*

3.1 Historical background

In 1543, Nicolaus Copernicus hypothesized that the planets revolve in circular orbits around the Sun. Subsequently, Tycho Brahe (1546–1601) carefully observed the locations of the planets (and 777) stars over a period of 20 years, using a sextant and compass. After his death, these observations were used by a student of his, Johannes Kepler, to deduce three empirical mathematical laws governing the orbit of one object around another. Kepler's third law for a planet orbiting the Sun is:

$$\frac{a^3}{T^2} = \text{constant} \quad (1)$$

where

- T is the orbital period (the planet's 'year'), and

- a is the length of the semi-major axis of the orbit*.

We now understand how Kepler's *purely empirical* laws can be derived from *physical* principles; in particular, his third law can be derived using Newton's theory of gravity. This means that Kepler's law can be applied not only to planets orbiting the Sun, but equally well to any two bodies in orbit. The generalized form of equation 1 is

$$M = K \frac{a^3}{T^2} \quad (2)$$

where K is a constant and M is the *sum* of the masses of the two bodies involved; however, in many cases, M is completely dominated by just one body. We can therefore often identify M with the mass of the body being orbited, to an excellent degree of approximation (for example, the mass of the Sun is far greater than the mass of any planet; and the mass of Jupiter is far greater than the mass of any of its moons).

By choosing appropriate units for M , a , and T , the constant K is equal to unity; those appropriate units are solar masses for M , (Earth) years for T , and the Astronomical Unit (AU; the mean Earth–Sun distance) for a . In other words, with these units, eqtn. 2 can be rewritten simply as $M = a^3/T^2$. Ask a demonstrator to show this is the case or try to prove it yourself with help.

3.2 Jupiter's Moons

The telescope was invented in or about 1609; shortly thereafter, Galileo used a telescope to discover Jupiter's four major moons.[†] The Jupiter system was especially important because it is like a miniature version of the solar system, with small bodies orbiting a much more massive central object. It provided Galileo with clear evidence that Copernicus' heliocentric model of the solar system was physically possible; not everything orbits the Earth. Unfortunately for Galileo, the church took issue with his findings and he was tried as a heretic.

4 Experiment

The 'Jupiter's Moons' program simulates the operation of a telescope equipped with a charge-coupled device (CCD) camera that delivers a digital image of the Jupiter system to your computer screen. The computer simulation is realistic in all important ways, and using it will give you a good feel for how astronomers collect data and control their telescopes.[‡]

You will 'observe' the four major, or Galilean, moons of Jupiter; in order of increasing distance from Jupiter, these are Io, Europa, Ganymede and Callisto[§] (sometimes identified

*All the planets and their moons in the solar system have orbits which are, to an excellent degree of approximation, elliptical (indeed, Kepler's first law is that the orbit of a planet about the Sun is an ellipse with the Sun at one focus). Most orbits have rather small eccentricities; if, as is assumed in this experiment, the orbit is circular, the the semi-major axis is the same as the radius of the orbit.

[†]Many smaller moons have been discovered since, and continue to be discovered; as of May 2003, the number was 60 – more than twice the official total in May 2002! The numbers change so rapidly that it's impossible to give 'the' number in this document; if you want to know today's total, check <http://www.ifa.hawaii.edu/sheppard/satellites/>.

[‡]In principle, you could use one of the ULO telescopes to *really* observe Jupiter's moons for many nights, but weather and lack of time prevent this. Instead, the computer simulation shows the moons as they would appear if you were to look through a telescope at the specified time.

[§]The order can be remembered using the mnemonic: 'I Eat Green Cabbage'. It is left as an exercise for the student to think up something more entertaining.

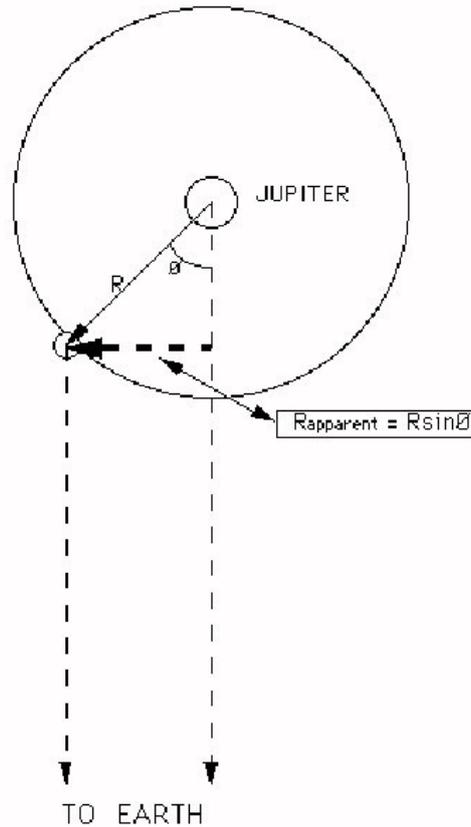


Figure 1: Geometry of Kepler's Third Law.

as moons I, II, III, IV). For an Earth-bound observer, the moons appear to be arranged nearly in a line because we are looking edge-on to the orbital plane of the moons of Jupiter. Thus one can only see the *projected* distance of the moon from Jupiter (see Figure 1).

This projection of the circular motion of the moons about Jupiter results in a back-and-forth movement (called Simple Harmonic Motion). In this type of motion the acceleration is proportional to the object's displacement (so that when a moon is near an extreme of its apparent, or projected, orbit it moves most slowly). Imagine the projected orbital plane of Jupiter's moons as the y -axis on a graph, with displacement eastwards from Jupiter being positive and westwards being negative. If you were to add an x -axis that measures time, a moon's orbital path is transformed from moving only up and down in the y direction to forming a sinusoidal curve (see Figure 2).

By taking sufficient measurements of the position of a moon, a sine curve can be fitted to the data and used to determine the radius of the orbit (the amplitude of the sine curve) and the period of the orbit (the period of the sine curve). Once a moon's orbital radius and period are known and converted into the appropriate units, Jupiter's mass may be determined, using eqtn. 2.

4.1 The 'Jupiter's Moons' Program

The PC should already be switched on. If the 'Jupiter Moons' program is not already running, find its icon and double-click the left mouse button to start the program. (In this script an unspecified 'click' means you should use the left mouse button; some of the features of the software require the use of the right mouse button and this will always be specified.)

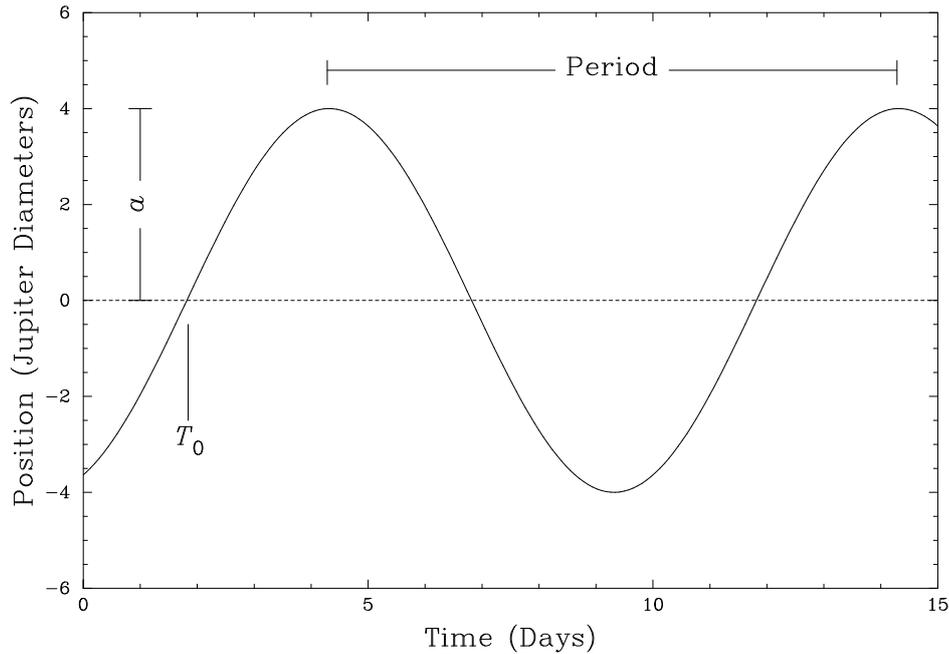


Figure 2: Graph showing illustrative data for an imaginary moon; its orbital period is $T = 10 \text{ d} = 0.0274 \text{ yr}$ (with $T_0 = 1.84$), and its semimajor axis is $a = 4D_{\text{J}} = 0.00381 \text{ AU}$.

From the menu bar (the options listed at the top of the first screen), select **File|LogIn**;¶ A ‘student accounting’ box will appear; just hit the OK button, without filling in any details (and confirm that this is okay when asked if this is what you want to do).

To begin the experiment click on **File|Run**. The next dialog box to appear is **Start Date and Time**. This should display today’s date. You can set the Universal Time|| to the time you are doing this experiment if you wish to view the moons’ positions as they would appear through a telescope at the time you’re working, but you can choose any time that suits you. When you are happy with the start date and time, click OK.

The moons will be displayed around Jupiter as they appear at the date and time shown in the bottom left-hand corner of the screen. Below the familiar calendar date and time, the Julian Date (JD) is displayed, which you’ll find much easier to use in your analysis**.

You can change the image scale by clicking on the ‘100×’, ‘200×’, ‘300×’ and ‘400×’ buttons at the bottom of the screen. In order to enhance the accuracy of your measurements of a moon’s position, you should use the largest possible magnification that leaves the moon on the screen.

You can also change the time step between ‘observations’. For Io, choose a time step of

¶This means click on the ‘File’ menu, then select ‘LogIn’ from the options offered. In this script, instructions in this font always indicate options on the computer screen.

||For most purposes, ‘Universal Time’ (UT) can be considered as the astronomical equivalent of Greenwich Mean Time (GMT).

**The JD system was devised by Joseph Justus Scaliger (who named it after his father, Julius) in 1582. It has a more-or-less arbitrary starting point such that JD 1 corresponds to January 1, 4713 BC, and increases by one each day thereafter (early dates can be expressed as negative numbers if necessary). The advantage of JDs is simply that the interval between any two arbitrary dates is much easier to calculate if the dates are expressed as Julian Dates (giving a simple subtraction) than if expressed as ordinary calendar dates (giving a horrible sum requiring that you know which years are leap years, the number of days in each month, when and how the switch from the Gregorian to Julian calendars occurred... yuck...).

about 4 hours, by clicking ‘File|Preferences|Timing’. (Other moons have longer orbital periods, and you will want to choose longer time intervals.)

If you wish, the moons can be displayed in different colours, to allow them to be differentiated more easily, by clicking ‘File|Preferences|ID Colours’. To facilitate your understanding of the actual motions of the moons, you can also activate an optional top view of the orbits, by clicking ‘File|Preferences|Top View’. Instead of displaying the satellite positions at pre-set intervals, you can also view their motions as an animation (File|Preferences|Animation; when ‘viewing’ the satellites, toggle the Cont. button to start and stop the animation).

4.2 Data Collection

You are now ready to make your measurements of the moons’ projected distances from Jupiter. To do this, move the mouse until the tip of the cursor arrow is centred on a moon. (Note that this can be a bit fiddly – you need to get the cursor tip almost exactly on the moon.) Then click the left mouse button. Information about the moon, including its name, will appear at the lower right-hand corner of the screen if you’re successful (if the moon’s name doesn’t appear, you’ve ‘missed’ with your mouse click – try again).

Other information displayed includes the x and y location on the screen (not used in this experiment), the projected distance (in units of Jupiter’s diameter, D_J) from the Earth–Jupiter line of sight for the selected moon, and an ‘E’ or ‘W’ to signify whether it is East or West of Jupiter. To record the position for subsequent analysis, click ‘Record Measurements’ and enter the moon’s distance from Jupiter (including ‘E’ or ‘W’ as appropriate, in upper or lower case) in the box beside its name. The time and date will automatically be recorded in this box. *Recording a moon as ‘E’ of Jupiter when it is actually ‘W’ (or vice versa) is the commonest data-taking error!*

Take measurements in this way at roughly regular intervals (by toggling the Cont. button) until sufficient data have been gathered to delineate the moon’s orbit accurately. You should aim to have at least a dozen measurements for each moon, distributed over at least one full orbit. It shouldn’t be necessary to take more than ~ 20 measurements for each moon. Your sampling frequency needs to be sufficiently high - say, at least 5 or 6 measures per orbit - to demonstrate the orbital characteristics clearly. (Don’t forget, this means measurements every ~ 4 hours or so for Io; longer intervals for other moons.) You will be able to measure the orbital period more accurately if your measurements are spread over 2–3 orbits, rather than a single orbit.

(*Tip:* For any given moon it’s a good idea to try to make perhaps 10 or so observations around one orbit, to define the broad orbital characteristics, and especially the amplitude, unambiguously; and then to add sparser observations over more orbits – up to 4–5 – to allow you to determine the period more precisely. The more orbits covered, the more accurately the period can be determined.)

There are several possible strategies for taking data:

- You can measure all four moons at each observing time;
- You can collect a complete set of measurements for one moon at a time;
- You can collect observations for a subset of moons at each observing time

The problem here is that the innermost moons orbit fastest, and so need to be observed at relatively short intervals; but using short intervals for the outer moons means that many measurements have to be made to cover the orbit. You will need to adopt the method that suits you best.

Note: Save your data at frequent intervals, by clicking ‘File|Data|Save’. You will need to give a filename that uniquely identifies the file as yours, and which you can remember (your surname or initials should do). You can reuse the same file name.

4.3 Data Analysis

Once you have recorded enough measurements, you can plot your results for a moon by clicking ‘File|Data|Analyze’. This will open a Data Analysis window. For example, if you wish to view the plot of your data for Callisto, click ‘Select|Callisto[IV]’ in this window. This plot gives the measured offset in Jupiter diameters as the y -axis and a modified Julian Date as the x -axis.^{††} You can view plots for the other moons by choosing their names under the **Select** menu.

4.3.1 Initial guesses

You should be able to make a rough estimate of a moon’s orbital period and semi-major axis simply by inspection. (You *may* find it helpful to connect your plotted points together. If you want to do this, click ‘Plot|Plot Type|Connect Points’.)

In order to determine these parameters more accurately, a sine curve can be fitted to the plot. To do this, click ‘Plot|Fit Sine Curve|Set Initial Parameters. You will need to provide initial estimates of the following parameters:

- T_0 [JD]: The Julian Date at which sine curve crosses the x -axis from beneath (see Figure 2);
- Period [Days]: The number of days to complete one orbit;
- Amplitude [D_J]: The maximum offset from the x -axis.

All these can be estimated by inspection, to one or two decimal places. (Note that there is a degree of ambiguity in choosing T_0 the value can be changed by an integer number of orbital periods and still be correct. The value you choose should coincide with one of the x -axis crossings in the time span of your data.) When your estimates have been entered, click ‘Ok’ to produce the sine curve.

4.3.2 Improving your results

You can now adjust the sine-curve parameters to achieve a ‘best fit’ to your data points. This is done by adjusting the scroll bars at the bottom of the screen. To fine tune the curve, click ‘Plot|Reset Scroll Bars|Fine Sensitivity’. (This is necessary to get the smallest error between the sine curve and your data points.)

The overall ‘goodness of fit’ is characterised by the ‘RMS Residual’ value given above the Cursor Position; the smaller the rms residual, the better the fit.* Use the scroll bars to adjust the sine-wave parameters until this number is as small as you can make it. To do this requires several iterations, adjusting each parameter in turn – i.e., make small changes to T_0 until the rms residual is as small as you can make it; then adjust the period similarly; then the amplitude; then go back and see if further adjustments to T_0 , then the period, then the amplitude, can reduce the rms residual; and so on.

You should certainly be able to achieve rms residuals smaller than $\sim 3\text{E-}01$ (where ‘E-01’ means ‘ $\times 10^{-1}$ ’; i.e., 3E-01 is equivalent to 0.3; similarly, ‘E-02’ means ‘ $\times 10^{-2}$ ’, and so on).

^{††}The Julian Date modified by subtracting the JD of your first observation – simply for convenience (otherwise unnecessarily large numbers are involved).

*If you don’t know what an ‘rms residual’ is, ask a demonstrator to explain.

If you are unable to achieve residuals as small as this, check your plots to see if there are any points which are obviously in error – this is very often the result of mixing ‘E’ and ‘W’ elongations. Correct, or remove, any seriously discrepant data points.

4.4 Printing Results

Once you have obtained a satisfactory sine-curve fit for a given moon, you can print the plot by clicking ‘Plot|Print Current Display’.[†] This should be done for each of the four moons. You should also generate a printout of your data table and the sine-wave parameters. To do this, first exit the data analysis window. Once back in the main window, click ‘File|Data|Print|Data Table’ for the data table and ‘File|Data|Print|Sine Parameters Summary’ for the sine parameters. The data table and the sine parameters each need to be printed only once, as they include results for all four moons.

You should now go on to complete the worksheet questions.

5 References and Background Reading

1. Zeilik, M. & Gregory, S. *Introductory Astronomy and Astrophysics*, (4th edition). The 3rd edition, by Zeilik, Gregory & Smith, is very similar.
2. Freedman, R.A. & Kaufmann, W.J., *Universe*, 7th edition; Chapters 4, especially §4–4 and 4–7

[†]The plots should come out on the multifunction printer/photocopier in the library, on the ground floor.

The Revolution of the Moons of Jupiter: Worksheet

It is always important to quote experimental results to sensible numbers of significant figures, and to quote the units in which your answers are given. Marks will be lost for inattention to these points in this experiment!

1. Having followed the instructions in Section 4, you should have 6 printouts (plots of observations and a fitted sine wave for each of four moons; a summary of observations; and a summary of sine fits). Write your name on each, and attach them to this worksheet when you submit it for marking.
2. For each moon, you should have values for the period, T , and the orbital semi-major axis (radius), a , in units of days for T and Jupiter diameters (D_{J}) for a . In order to use Kepler's Third Law (in the form of eqn. 2), you need to convert the period into years (by dividing by 365.25: the number of days in a year: 365.25), and the orbital radius into AU (by dividing by 1050: the number of Jupiter diameters in an astronomical unit).

Enter your values in the table below, and use equation 2 to calculate the mass of Jupiter (M_{J}) from the results for each moon.

Calculate the average mass of Jupiter, from your results from the 4 moons. *Be sure to quote your result to a sensible number of significant figures* – you should be able to gauge this from the dispersion in the results.

Moon	T (d)	T (yr)	a (D_{J})	a (AU)	M_{J} (M_{\odot})
Io					
Europa					
Ganymede					
Callisto					
Mean:	

3. The orbit of Earth's moon has a period of 27.3 days and semi-major axis of 2.56×10^{-3} AU ($= 3.84 \times 10^5$ km). By using eqtn. 2, determine the mass of the Earth in solar units, and in kg. (Show your work; the mass of the Sun is 1.989×10^{30} kg.)

4. Using your earlier results for the mass of Jupiter and the mass of the Earth, express the mass of Jupiter in units of the Earth's mass.

5. There are moons beyond the orbit of Callisto. Will they have longer or shorter periods than Callisto? Why?

6. Which do you think would cause the larger error in mass: a 10% error in T or a 10% error in a ? Why?

7. In principle, your measured values for the orbital periods of the Galilean satellites will not be *exactly* correct – even if you made absolutely perfect measurements. Can you think why this might be? (Hint: Olaus Rømer discovered the finite speed of light from observations of the Galilean satellites, by noting that the *apparent* orbital periods varied with time.)